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Failure Detection of Dynamical Systems with the State Chi-Square Test

Ren Da*
Technion—Israel Institute of Technology, Haifa 32000, Israel

Two novel approaches are developed to improve the failure sensitivity of the failure detection systems in which the chi-square test is used to monitor the state estimates of the Kalman filter. The first involves the use of two state propagators. These two state propagators are alternatively reset with the data of the Kalman filter to increase their accuracy and thereby the failure sensitivity of the detection system. By choosing a suitable time interval between resetting a state propagator and using it for the failure detection, the risk of using a contaminated state propagator as a failure detection reference is avoided. The second approach involves monitoring the state estimates of the Kalman filter individually. With this approach the test statistics related to the state estimates most seriously affected by the failures are utilized to enhance failure sensitivity of the detection system. The effectiveness of the proposed approaches is demonstrated in the failure detection problem of a global positioning system/strapdown inertial navigation system integrated navigation system. The simulation results show that the failure sensitivity of the detection system is remarkably increased.

I. Introduction

To maintain a high level of performance in a dynamical system, it is necessary that failures be detected and isolated promptly so that appropriate remedies can be applied. Although catastrophic or hard over failures can be uncovered rapidly by online built in testing, more subtle or soft failures can only be detected by the use of more sophisticated techniques based on error estimation/decision theory. Over the past decades, numerous approaches to the detection and isolation of soft failures in dynamical systems have been developed,[1–11] but most of them are suitable only for low-order or time-invariant dynamical systems.[2]

As a statistical hypothesis testing method for examining whether or not a random Gaussian vector has the assumed mean and covariance, the chi-square test is widely applied for detecting soft failures in dynamical systems.[1–4] Assume $\xi(k)$ is a p-dimensional random Gaussian white vector which has zero mean and covariance $V(k)$ under normal working status. From the fact that

$$\zeta_\xi(k) = \sum_{i=k-N+1}^{k} \xi^T(i)V^{-1}(i)\xi(i) \sim \chi^2(Np) \tag{1}$$

i.e., $\zeta_\xi(k)$ is a chi-square random variable with Np degree of freedom (N is the window length), the following rule can be considered for the failure detection[4]:

$$\begin{array}{lll} \text{if} & \zeta_\xi(k) \geq \varepsilon_\xi & \text{then there is a failure} \\ \text{if} & \zeta_\xi(k) < \varepsilon_\xi & \text{then there is no failure} \end{array} \tag{2}$$

where ε_ξ is the chosen decision threshold. With the aid of a chi-square distribution table,[12] the probability of false alarm can be calculated as a function of the window length N and the decision thresholds ε_ξ. The important thing in the design of a failure detection system with the application of the chi-square test is to find a suitable Gaussian vector ξ, which is sensitive to system failures and whose mean and covariance are known or can be easily calculated.

The chi-square test has been applied to monitor the residual of the Kalman filter by many investigators,[1,4] and also suggested to check the consistency of the outputs of several navigation sensors and/or the Kalman filters.[2] An interesting application of the chi-square test for failure detection was proposed in Ref. 3, where a state propagator was used to provide a reference system for the failure detection (an idea originally proposed in Ref. 5), and the chi-square test was used to test the consistency of the state estimate of the Kalman filter and that of the state propagator. The approach is named the state chi-square test (SCST) in the paper because the chi-square test is used to monitor the state estimate of the Kalman filter instead of the residual. An example of the usefulness of the SCST was initially shown when it was used to detect the accelerometer failures in an inertial navigation system/Doppler integrated navigation system.[3]

In this paper, special attention is focused on increasing the failure sensitivity of the detection systems where the SCST discussed in Ref. 3 is used. First the working principle of the SCΩST is briefly described. Then two problems which greatly affect the failure sensitivity of the current SCST are analyzed. After that two approaches are developed to improve the failure sensitivity of the SCST. Finally the usefulness of the approaches is demonstrated when they are applied to the soft failure detection of a global positioning system (GPS)/strapdown inertial navigaton system (SDINS) integrated navigation system.

II. State Chi-Square Test

Consider the following error model for a discrete-time dynamical system[1]:

$$\boldsymbol{x}(k+1) = \Phi(k)\boldsymbol{x}(k) + \boldsymbol{b}\delta_{k\theta} + \Gamma(k)\boldsymbol{w}(k) \tag{3}$$

$$\boldsymbol{y}(k) = H(k)\boldsymbol{x}(k) + \boldsymbol{c}\delta_{k\phi} + \boldsymbol{v}(k) \tag{4}$$

$$E[\boldsymbol{w}(k)] = 0, \qquad E[\boldsymbol{w}(k)\boldsymbol{w}(j)] = Q(k)\delta_{kj} \tag{5}$$

$$E[\boldsymbol{v}(k)] = 0, \qquad E[\boldsymbol{v}(k)\boldsymbol{v}(j)] = R(k)\delta_{kj} \tag{6}$$

where $\boldsymbol{x}(k)$ is an n-dimensional state, with Gaussian initial condition $\boldsymbol{x}(0)$, which has mean $\boldsymbol{x}_0$ and covariance P_0. Here δ_{ij} is a Kronecker function (it is 1 when $i = j$ and 0 otherwise), and $\boldsymbol{x}(0)$, $\boldsymbol{w}(k)$, and $\boldsymbol{v}(k)$ are assumed to be statistically independent. $Q(k)$ is positive- semidefinite and $R(k)$ is positive-definite. The vectors $\boldsymbol{b}$ and $\boldsymbol{c}$ represent the unknown failures; θ and ϕ represent the unknown time of failures.

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*Postdoctoral Fellow, Faculty of Aerospace Engineering; currently Senior Researcher, American GNC Corporation, Chatsworth, CA.

If no failure occurs ($\theta = \phi = \infty$), the following standard Kalman filter algorithm gives the optimal estimate $\hat{x}_K(k \mid k)$ of the system state $x(k)$ (the subscript K denotes the Kalman filter):

$$\hat{x}_K(k+1 \mid k) = \Phi(k)\hat{x}_K(k \mid k) \tag{7}$$

$$P_K(k+1 \mid k) = \Phi(k)P_K(k \mid k)\Phi^T(k) + \Gamma(k)Q(k)\Gamma^T(k) \tag{8}$$

$$\hat{x}_K(k+1 \mid k+1) = \hat{x}_K(k+1 \mid k) + K(k+1)\gamma(k+1) \tag{9}$$

$$\gamma(k+1) = y(k+1) - H(k+1)\hat{x}_K(k+1 \mid k) \tag{10}$$

$$U(k+1) = H(k+1)P_K(k+1)H^T(k+1) + R(k+1) \tag{11}$$

$$K(k+1) = P_K(k+1)H^T(k+1)U^{-1}(k+1) \tag{12}$$

$$P_K(k+1 \mid k+1) = [I - K(k+1)H(k+1)]P_K(k+1 \mid k)$$
$$\times [I - K(k+1)H(k+1)]^T + K(k+1)R(k+1)K^T(k+1) \tag{13}$$

with the initial conditions:

$$\hat{x}_K(0|0) = x_0 \qquad P_K(0|0) = P_0 \tag{14}$$

Because the residual $\gamma(k)$ is a Gaussian random white vector with zero mean and covariance $U(K)$ for the optimal Kalman filters,[13] failure detection can then be performed by monitoring the residual $\gamma(k)$ with the chi-square test.[1,4] The implementation of the chi-square test with the residual $\gamma(k)$ is quite simple, but with rather severe limitations on the performance.[1]

Instead of monitoring the residual, the SCST detects system failures by monitoring the state estimate of the Kalman filter with the chi-square test.[3] To monitor the state estimate of the Kalman filter, a state propagator is used as a reference system, whose state estimate $\hat{x}_S(k)$ and covariance matrix $P_S(k)$ (the subscript S denotes the state propagator) are propagated based only on the a priori system model information:

$$\hat{x}_S(k+1) = \Phi(k)\hat{x}_S(k) \tag{15}$$

$$P_S(k+1) = \Phi(k)P_S(k)\Phi^T(k) + \Gamma(k)Q(k)\Gamma^T(k) \tag{16}$$

with the initial conditions:

$$\hat{x}_S(0) = x_0, \qquad P_S(0) = P_0 \tag{17}$$

Define the estimation error of the Kalman filter and that of the state propagator as:

$$e_K(k \mid k) = x(k) - \hat{x}_K(k \mid k) \tag{18}$$

$$e_S(k) = x(k) - \hat{x}_S(k) \tag{19}$$

then the difference between the estimate of the Kalman filter and that of the state propagator

$$\beta(k) \triangleq \hat{x}_K(k) - \hat{x}_S(k \mid k) = e_S(k) - e_K(k \mid k) \tag{20}$$

is a Gaussian random vector with zero mean and covariance $B(k)$. Under the condition of

$$\hat{x}_K(0|0) = \hat{x}_S(0) = x_0 \tag{21}$$

$$P_K(0|0) = P_S(0) = P_0 \tag{22}$$

$B(k)$ can be calculated by the following equation:[3]

$$B(k) \triangleq E[\beta(k)\beta^T(k)] = P_S(k) - P_K(k \mid k) \tag{23}$$

and so

$$\zeta_\beta(k) \triangleq \beta^T(k)B^{-1}(k)\beta(k) \sim \chi^2(n) \tag{24}$$

where n is the dimension of $\beta(k)$. Therefore the following rule can be considered for the failure detection

$$\begin{aligned} &\text{if} \quad \zeta_\beta(k) \geq \varepsilon_\beta \quad \text{then there is a failure} \\ &\text{if} \quad \zeta_\beta(k) < \varepsilon_\beta \quad \text{then there is no failure} \end{aligned} \tag{25}$$

where ε_β is the chosen threshold.

Direct computation of $B^{-1}(k)$ in Eq. (24) can be avoided by computing the Cholesky decomposition $B(k) = L(k)L^T(k)$ and solving a triangular system of equations.[3]

III. Two Problems in the State Chi-Square Test

In the current SCST, there are two problems which may greatly influence its detection sensitivity to system failures if not carefully considered.

A. Problem 1

For an optimal Kalman filter, the state estimate accuracy usually increases with filtering time, and the influences of the initial errors and the process noises on the filter performance are greatly suppressed because of the measurement update. But the influences of the initial errors and process noises on the accuracy of the state propagator are much greater than the Kalman filter, because the state propagator propagates the state estimates based only on the a priori system model information. Then, while the accuracy of the Kalman filter increases and its covariance $P_K(k \mid k)$ reaches small/bounded value with the increasing k, the accuracy of the state propagator decreases and its covariance $P_S(k)$ keeps on increasing, and is eventually much larger than $P_K(k \mid k)$.

In the design of a failure detection system, one of the most important things is its sensitivity to failures. If a small failure makes a large increase in the test statistic ζ, it means that the failure detection system is sensitive to the failure. From Eqs. (23) and (24), it is clear that the failure sensitivity of the SCST degrades when the accuracy of the state propagator decreases, because $B(k)$ increases with the covariance P_S of the state propagator. The following example helps us further look into this problem.

Example 1: Assume a single-input, single-output system:

$$x(k+1) = ax(k) + w(k) + b\delta_{k\theta} \tag{26}$$

$$y(k) = hx(k) + v(k) + c\delta_{f\phi} \tag{27}$$

$$E[w(k)] = 0, \qquad E[w(k)w(j)] = q\delta_{kj} \tag{28}$$

$$E[v(k)] = 0, \qquad E[v(k)v(j)] = r\delta_{kj} \tag{29}$$

with Gaussian initial condition $x(0)$, which has mean x_0 and covariance p_0; $x(0)$, $w(k)$, and $v(k)$ are assumed to be statistically independent; $q \geq 0$ and $r > 0$; b and c are the unknown failures; and θ and ϕ are the unknown time of failures.

Let $e_{K,0}(k \mid k)$ and $e_K(k \mid k)$ denote, respectively, the state estimation error of the Kalman filter when there is no failure and that of the Kalman filter when a failure occurs. Then their difference $\Delta e_K(k \mid k)$

$$\Delta e_K(k \mid k) \triangleq e_K(k \mid k) - e_{K,0}(k \mid k) \tag{30}$$

reflects the effect of the failure on the state estimation error of the Kalman filter. Using $\zeta_{\beta,0}(k)$ and $\zeta_\beta(k)$ to represent the test statistics with and without the failure, and $\Delta\zeta_\beta(k)$ to represent the change of test statistic that results from the failure, we have, from Eq. (24):

$$\zeta_{\beta,0}(k) = \frac{[e_{K,0}(k|k) + \Delta e_K(k|k) - e_S(k)]^2}{p_S(k) - p_K(k|k)} \tag{31}$$

$$\zeta_\beta(k) = \frac{[e_{K,0}(k|k) - e_S(k)]^2}{p_S(k) - p_K(k|k)} \tag{32}$$

and

$$\Delta\zeta_\beta(k) = \zeta_\beta(k) - \zeta_{\beta,0}(k)$$

$$= \frac{2\,[e_{K,0}(k|k) - e_S(k)\,]\,\Delta e_K(k|k) + [\Delta e_K(k|k)\,]^2}{p_S(k) - p_K(k|k)}$$

$$= 2\zeta_{\beta,0}(k)\frac{\Delta e_K(k|k)}{e_{K,0}(k|k) - e_S(k)} + \frac{[\Delta e_{K,0}(k|k)\,]^2}{p_S(k) - p_K(k|k)} \tag{33}$$

Equation (33) shows that $\Delta\zeta_\beta(k)$ depends on the ratio of $\Delta e_K(k \mid k)$ to the difference between $e_{K,0}(k \mid k)$ and $e_S(k)$, and the ratio of $[\Delta e_K(k \mid k)]^2$ to the difference between $p_S(k)$ and $p_K(k \mid k)$. Therefore the degradation of the accuracy of the state propagator, which results in the increase of the difference between $e_{K,0}(k \mid k)$ and $e_S(k)$ and the difference between $p_S(k)$ and $p_K(k \mid k)$, greatly damages the sensitivity of the SCST to the failure. In other words, the failure may not be detected if the state propagator is very inaccurate.

Assume the initial errors and system parameters in example 1 are

$$a = h = 1, \qquad q = r = 0.01 \tag{34}$$

$$x_K(0|0) = x_S(0) = x_0 = 0 \tag{35}$$

$$p_K(0|0) = p_S(0) = p_0 = 1 \tag{36}$$

The covariances $p_K(k \mid k)$ of the Kalman filter and $p_S(k)$ of the state propagator are presented in Fig. 1. It shows that while $p_K(k \mid k)$ approximates a small stable value (about 0.0062) in a few filtering steps, $p_S(k)$ keeps on increasing. The state propagator is very inaccurate compared to the Kalman filter. Assume a system failure (b = 4, c = 0) occurs at θ = 50. Figure 2 shows that the SCST fails because the test statistic $\zeta_1(k)$ of SCST is not big enough to pass the chosen threshold ε = 7.879 (corresponding to a false alarm probability of 0.005).

B. Problem 2

The effects of failures on each state estimate may be quite different. Some states will be affected by a failure more directly than other states. The test statistic ζ_β calculated by Eq. (24) reflects only the overall effects of a failure on the system state estimates, but not the effects on those states which are most seriously affected by the failure. The decision threshold ε_β with a chosen false alarm probability increases with the dimension of $\beta(k)$. For example, ε = 7.879 when dim(β) = 1 and ε = 41.4010 when dim(β) = 21 (with false alarm probability 0.005). Therefore, for large dynamical systems with many state variables, the detection rule of Eq. (25), which detects failures by comparing the test statistic ζ_β with the decision threshold ε_β, may be not suitable. The detection of failures which affect only a few system states may be greatly delayed or not occur.

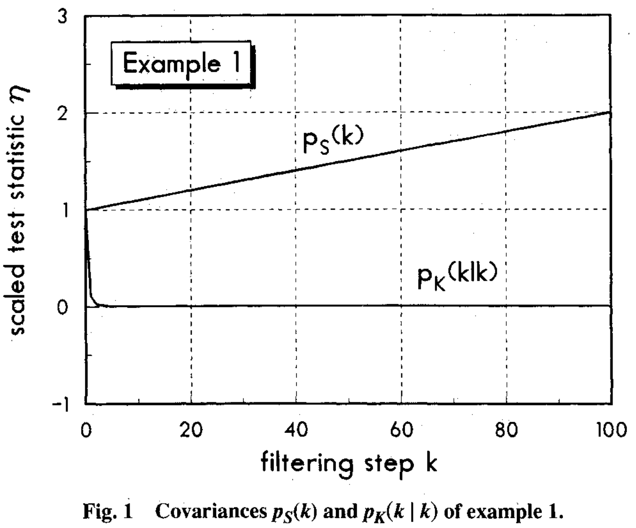

Fig. 1 Covariances $p_S(k)$ and $p_K(k \mid k)$ of example 1.

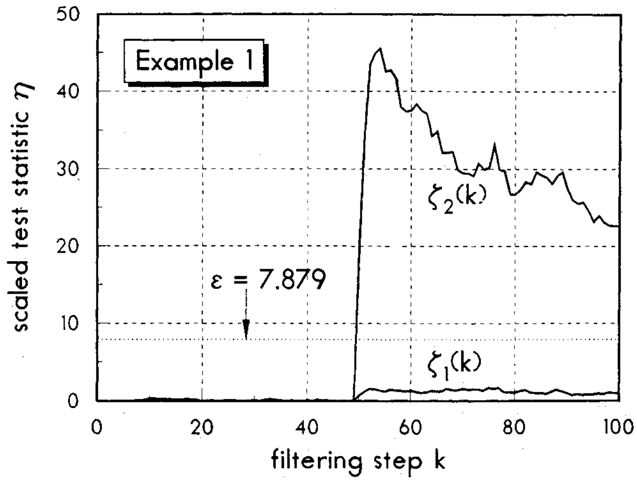

Fig. 2 Comparison of the test statistics of the two SCST approaches.

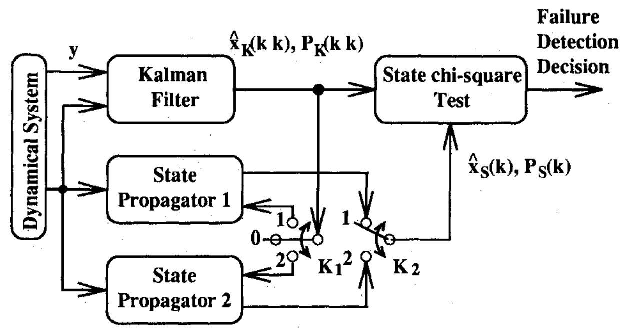

Fig. 3 Failure detection structure for dynamical systems.

IV. Modifications of the State Chi-Square Test

In this section some approaches are proposed to overcome the problems presented in the preceding section.

A. Approach for Problem 1

It is known that the accuracy of the state propagator is very important to the failure sensitivity of the SCST. Based on the fact that the estimation accuracy of the Kalman filter is normally much higher than that of the state propagator, it is reasonable to consider using the estimation results of the Kalman filter to reset the state propagator regularly.

However, because there is usually a time interval between the time when a failure happens and when it is detected, the possibility exists that the state propagator may be contaminated by the Kalman filter data if an undetected failure has already happened. To avoid the situation of using a contaminated state propagator as the failure detection reference, two state propagators are suggested in our failure detection system, as shown in Fig. 3. The two state propagators are reset with the Kalman filter data and used as the failure detection reference alternatively. Assume t_j (j = 1, ...) are the time points at which one of the state propagators is reset with the Kalman filter data and the other is put to work as the failure detection reference. Then the switch K_1 will be at position 1 to reset the state propagator 1 when $k = t_{2i}$ (i = 1, ...), at position 2 to reset the state propagator 2 when $k = t_{2i-1}$, and at position 0 when $k \neq t_j$ (j = 1, ...). The switch K_2 will be at position 1 to use the state propagator 1 as the failure detection reference when $t_{2i-1} \leq k < t_{2i}$ and at position 2 to use the state propagator 2 as the failure detection reference when $t_{2i} \leq k < t_{2i+1}$. With this method, a newly reset state propagator does not work as the failure detection reference right away. Only after a certain period of time $\Delta t = t_{j+1} - t_j$, when it is certain no failure occurred before the state propagator was reset, will it be put to work. In this way, the risk of using a contaminated state propagator as the failure detection reference can be greatly avoided.

The choice of the suitable time interval Δt is important, depending on the real system design requirements. The time interval Δt should be long enough to allow a failure to be detected before the state propagator contaminated by the failure is put to work. But it should also not be so long that the high accuracy of the state propagator is compromised.

Using the same system model and failure assumption as that given in example 1, and choosing the time interval Δt as 20 filtering steps, the test statistic $\zeta_2(k)$ in Fig. 2 shows the effectiveness of the proposed approach. The failure is detected promptly. The change of the test statistic $\zeta_2(k)$ resulting from the failure is 20 times greater than that of test statistic $\zeta_1(k)$.

B. Approaches for Problem 2

The following two methods are considered to overcome problem 2.

Method 1: Let

$$\alpha(k) = B^{-1/2}(k)\beta(k) \tag{37}$$

where $B^{-1/2}(k)$ is the square root matrix of $B(k)$. Then from the fact that

$$E[\alpha(k)] = 0 \tag{38}$$

$$E[\alpha(k)\,\alpha^T(k)] = B^{-1/2}(k)E[\beta(k)\beta^T(k)]\,B^{-1/2}(k) = I \tag{39}$$

where I is the identity matrix, it is known that the components of $\alpha(k)$ are independent normal Gaussian variables. Considering that the effects of a failure on the components of $\alpha(k)$ are usually quite different, the components of $\alpha(k)$ can be tested individually by the following rule:

$$\begin{aligned} &\text{if } \zeta_{\alpha i}(k) \ge \varepsilon_{\alpha i} && \text{then there is a failure} \\ &\text{if } \zeta_{\alpha i}(k) < \varepsilon_{\alpha i} && \text{then there is no failure} \end{aligned} \tag{40}$$

or equally

$$\begin{aligned} &\text{if } \eta_{\alpha i}(k) \ge 1 && \text{then there is a failure} \\ &\text{if } \eta_{\alpha i}(k) < 1 && \text{then there is no failure} \end{aligned} \tag{41}$$

where $\zeta_{\alpha i}(k) \triangleq \alpha_i^2(k)$ scaled test statistic $\eta_{\alpha i}(k) \triangleq \zeta_{\alpha i}(k)/\varepsilon_{\alpha i}$. Then it is expected that the test statistics that are most sensitive to the failure will flag the failure first.

For the failures which significantly affect two or more components of $\alpha(k)$ at the same time, testing two or more components of $\alpha(k)$ together may result in a quicker failure detection than testing the components of $\alpha(k)$ individually. To test two or more components of $\alpha(k)$ together is easy because the components of $\alpha(k)$ are independent Gaussian variables, and so the square sum of arbitrary m components of $\alpha(k)$ is a chi-square variable with m degree of freedom. Actually, when $m = n$, we have the same detection rule as Eq. (25).

Method 2: An easier way to overcome problem 2 is to test the components of $\beta(k)$ individually. From

$$E[\beta_i(k)] = 0 \tag{42}$$

$$E[\beta_i^2(k)] = B_{i,i}(k) \tag{43}$$

where $\beta_i(k)$ are the components of $\beta(k)$ and $B_{i,i}(k)$ the diagonal elements of covariance matrix $B(k)$, we have

$$\zeta_{\beta i} \triangleq \frac{\beta_i^2(k)}{B_{i,i}(k)} \sim \chi^2(1) \tag{44}$$

Therefore the components of $\beta(k)$ can be tested individually with the following detection rule:

$$\begin{aligned} &\text{if } \zeta_{\beta i}(k) \ge \varepsilon_{\beta i} && \text{then there is a failure} \\ &\text{if } \zeta_{\beta i}(k) < \varepsilon_{\beta i} && \text{then there is no failure} \end{aligned} \tag{45}$$

or equally

$$\begin{aligned} &\text{if } \eta_{\beta i}(k) \ge 1 && \text{then there is a failure} \\ &\text{if } \eta_{\beta i}(k) < 1 && \text{then there is no failure} \end{aligned} \tag{46}$$

where $\eta_{\beta i} \triangleq \zeta_{\beta i}(k)/\varepsilon_{\beta i}$.

With the given detection rule, we can expect that the test statistics of the components $\beta(k)$, which are most seriously affected by the failures, will flag the failures first. If it is necessary, method 2 can also be extended to test two or more closely related components of $\beta(k)$ together. For example, if $\beta_i(k)$ and $\beta_j(k)$ are required to be tested together, we have

$$\zeta_{\beta ij} = [\beta_i(k)\,\beta_j(k)] \begin{bmatrix} B_{i,i}(k) & B_{i,j}(k) \\ B_{j,i}(k) & B_{j,j}(k) \end{bmatrix}^{-1} \begin{bmatrix} \beta_i(k) \\ \beta_j(k) \end{bmatrix} \sim \chi^2(2) \tag{47}$$

Actually, the detection rule of Eq. (25) also means that all of the components of $\beta(k)$ are tested together.

V. Failure Detection of a Global Positioning System/ Strapdown Inertial Navigation System

In this section, the approaches proposed for improving the failure sensitivity of the SCST are applied to the failure detection problem of a GPS/SDINS integrated navigation system. The chosen error states of the GPS/SDINS integrated navigation system are as follows:

$$\boldsymbol{x}(t) = [\delta r_N, \delta r_W, \delta r_Z, \delta v_N, \delta v_W, \delta v_Z, \Phi_N, \Phi_W, \Phi_Z, \Delta_x, \Delta_y, \Delta_z, \varepsilon_x, \varepsilon_y, \varepsilon_z, \delta r_{GN}, \delta r_{GW}, \delta r_{GZ}, \delta v_{GN}, \delta v_{GW}, \delta v_{GZ}]^T \tag{48}$$

where the subscript N, W, Z represents the north-west-up local level reference frame, and x, y, z the aircraft body reference frame. Here, δr_N, δr_W, δr_Z are the SDINS position errors; δv_N, δv_W, δv_Z are the SDINS velocity errors; and Φ_N, Φ_W, Φ_Z are the SDINS attitude errors and azimuth error. The accelerometer, gyro, GPS position, and GPS velocity errors are all modeled as the first-order Markov process plus the random white process. The correlation time constants of the Markov processes of the accelerometer, gyro, and GPS errors are chosen, respectively, as 60, 30, and 15 min. The measurements of the Kalman filter are the differences between the position data from the GPS receiver and those from the SDINS, as well as the differences between the velocity data from the GPS receiver and those from the SDINS. The model parameters used in the simulation analysis are listed in Table 1, where the effects of the selective availability in the newer block II GPS satellites on the

Table 1 Model parameters

Error source	Magnitude (1σ)
SDINS initial position errors δr_N, δr_W, δr_Z	100.0 m
SDINS initial velocity errors δv_N, δv_W, δv_Z	1.0 m/s
SDINS initial attitude errors Φ_N, Φ_W	300.0 arcsec
SDINS initial azimuth error Φ_Z	900.0 arcsec
SDINS accelerometer Markov process Δ_x, Δ_y, Δ_z	100.0 μg
SDINS accelerometer random white process	1.0 $\mu g * s^{1/2}$
SDINS gyro drift Markov process ε_x, ε_y, ε_z	0.1 deg/h
SDINS gyro drift random white process	0.001 deg/h$^{1/2}$
GPS position Markov process δr_{GN}, δr_{GW}, δr_{GZ}	100.0 m
GPS position random white noise sequence	40.0 m
GPS velocity Markov process δv_{GN}, δv_{GW}, δv_{GZ}	0.1 m/s
GPS velocity random white noise sequence	0.005 m/s

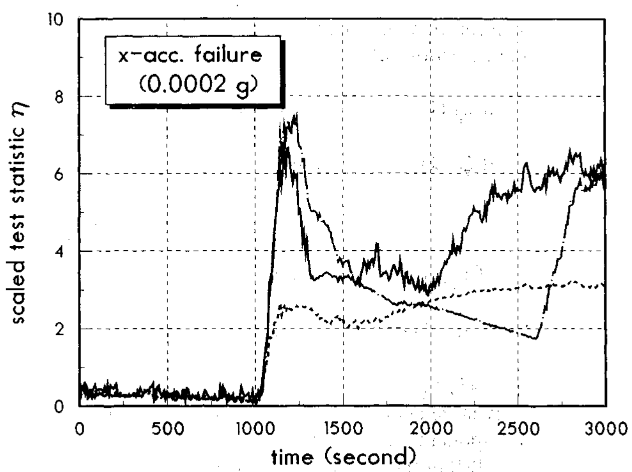

Fig. 4 SCST results of the three methods (x-accelerometer failure).

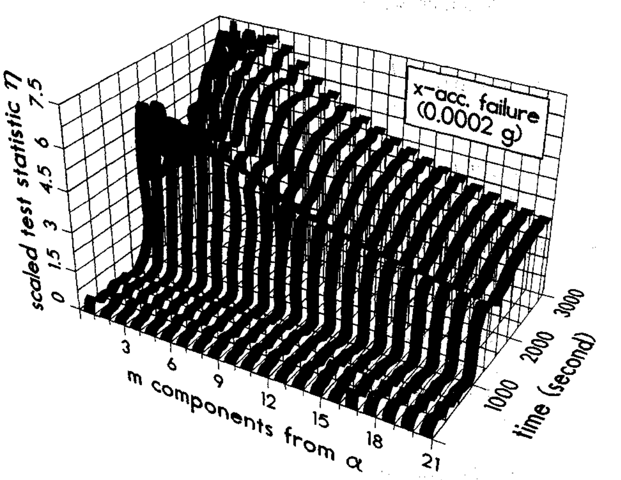

Fig. 5 SCST results when m components from α are tested together.

accuracy of the GPS position and velocity measurements have been considered.[14] The effectiveness of the proposed approaches is investigated under one of the following assumed system failures: Case 1: the measurement error of the x-accelerometer jumps $2 \times 10^{-4}g$ ($g = 9.8$ m/s^2); and case 2: the measurement error of the x-gyro jumps 0.5 deg/h. The failures are supposed to happen at $t = 1000$ s. All of the detection thresholds ε are chosen corresponding to a false alarm probability of 0.005. For example, $\varepsilon = 7.879$ when the components of $\alpha(k)$ and $\beta(k)$ are tested individually and $\varepsilon = 41.4010$ when $\beta(k)$ is tested as a vector ($n = 21$). To make it easier to compare the results of the different approaches, the scaled test statistic $\eta = \zeta/\varepsilon$ is used in the simulation figures.

The mission scenario was 300 s of flight due west, followed by a 90 deg turn to the north. Flight speed was held at 300 m/s. As presented in Fig. 3, two state propagators are used in the GPS/SDINS failure detection system. They are reset with the data from the Kalman filter and used as the reference system for the failure detection alternatively. The time interval between resetting a state propagator with the Kalman filter data and using it as the failure detection reference is 400 s (when the state propagators were not reset with the Kalman filter data, the failures in case 1 and case 2 were not detected and therefore not shown).

Figures 4–6 represent the simulation results for failure case 1. Figure 4 represents the test statistic when the whole vector $\beta(k)$ is tested with the rule of Eq. (25) (dashed line), the maximum test statistic when the components of $\alpha(k)$ are tested individually with method 1 (solid line), and the maximum test statistic when the components of $\beta(k)$ are tested individually with method 2 (chain-dotted line). Figure 5 shows the maximum test statistics when the m ($m = 1, \ldots, n$) components chosen from $\alpha(k)$ are tested together [the x axis represents the number of the tested components chosen from $\alpha(k)$]. Figure 6 presents the test statistics of the components of $\beta(k)$ when they are tested individually [the x axis represents the sequence number of the state variables in Eq. (48)].

Figures 7–9 represent the simulation results for failure case 2. As in Fig. 4, the dashed line in Fig. 7 represents the test statistic of the vector $\beta(k)$, the solid line represents the maximum test statistic of the components of $\alpha(k)$, and the chain-dotted line represents the maximum test statistic of the components of $\beta(k)$. Figure 8 shows the maximum test statistics when m ($m = 1, \ldots, n$) components chosen from $\alpha(k)$ are tested together and Fig. 9 shows the test statistics of the components of $\beta(k)$ when they are tested individually.

As we expected, the failures affect different test statistics very differently, as seen in Figs. 6 and 9. Therefore, the failure sensitivity of the detection system is greatly enhanced when the components of $\alpha(k)$ or $\beta(k)$ are tested instead of the vector of $\beta(k)$, as illustrated in Figs. 4 and 7. Figures 5 and 8 show that the failure sensitivity generally decreases when more components chosen from $\alpha(k)$ are tested together. Based on the simulation results, the advantages of the proposed approaches can be summarized as follows.

A. Short Detection Time

When the components of $\alpha(k)$ or $\beta(k)$ are tested individually with the detection rules of Eq. (41) or Eq. (46), we get faster failure detection than the detection rule of Eq. (25). It takes about 500 s to flag the x-gyro failure when whole vector $\beta(k)$ is tested with the detection rule of Eq. (25), but it takes, respectively, only about 200 or 300 s when the components of $\alpha(k)$ or $\beta(k)$ are tested individually with Eq. (41) or Eq. (46). The effects of the failures to the system state estimates are quite different (as seen in Figs. 8 and 9). When the components of $\alpha(k)$ or $\beta(k)$ are tested individually, the

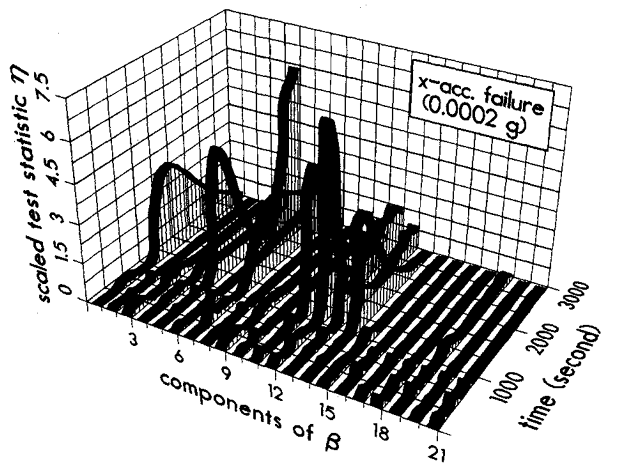

Fig. 6 SCST results when the components of β are tested individually.

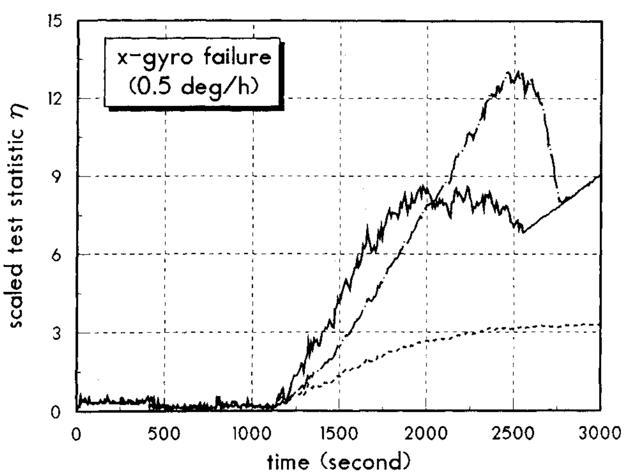

Fig. 7 SCST results of the three methods (x-gyro failure).

information from the state estimates affected most seriously by the failures is utilized to detect the failures.

B. High Failure Sensitivity

When the components of $\alpha(k)$ or $\beta(k)$ are tested, the sensitivity of the detection system to the failures is also much higher than when the whole vector $\beta(k)$ is tested. Figures 4 and 7 show that the maximum scaled test statistics, when the components of $\alpha(k)$ and $\beta(k)$ are tested individually, are two to three times larger than the scaled test statistics when the whole vector of $\beta(k)$ is tested. Some components of $\alpha(k)$ or $\beta(k)$ are evidently more sensitive to the failures than other components. In Fig. 9, for example, the test statistics of η_4, η_{10}, and η_{14} change greatly because the x-gyro failure results in large estimation errors to Φ_N, ε_x, and Δ_y. On the other hand, the failure has little effect on the state estimates of the GPS measurement errors, so their test statistics remain unchanged.

C. More Failure Isolation Information

For real dynamical systems, failure isolation is very important. In the GPS/SDINS integrated navigation system, it must be determined whether the failure is from the GPS or from the SDINS. But if the vector $\beta(k)$ is tested with the detection rule of Eq. (25), no further information for the failure isolation is given. If the components of α and β are considered as the tested variables, more information for the failure isolation can be obtained by analyzing the relationship between the failures and the tested variables. For example, if the test statistics corresponding to the GPS error variables are very small when the failure detection system flags, it is reasonable to decide the failure is most likely from the SDINS.

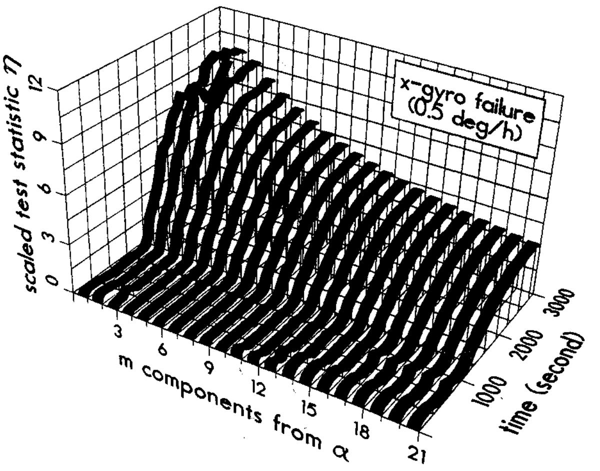

Fig. 8 SCST results when m components from α are tested together.

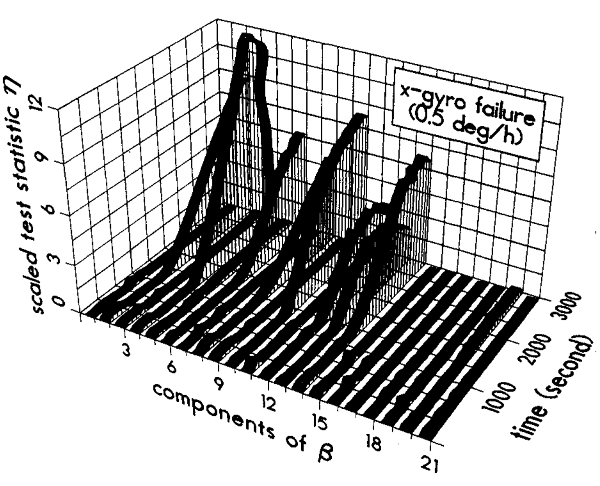

Fig. 9 SCST results when the components of β are tested individually.

On-line diagnosis may also be possible by carefully analyzing the relationship between the failures and the test statistics.

VI. Conclusions

In this paper a number of issues involved in improving the failure sensitivity of the detection systems where the state chi-square test (SCST) is used to monitor the state estimates of the Kalman filter have been discussed. Two existing problems in the current SCST were analyzed and then two approaches were proposed to overcome them.

A. Using Two State Propagators

Two state propagators are used in the failure detection system. The state propagators are alternatively reset with the data of the Kalman filter to increase their accuracy and thereby the detection sensitivity of the system. By carefully choosing the reset time, the risk of using a contaminated state propagator as a reference system is avoided.

B. Testing the Components of $\alpha(k)$ or $\beta(k)$ Individually

Testing the components of $\alpha(k)$ or $\beta(k)$ individually can utilize the information given by the state estimates which are most seriously affected by the failures, so the detection sensitivity of the SCST is greatly enhanced. If it is necessary, two or more components of $\alpha(k)$ or $\beta(k)$ can be tested together to improve failure detection time.

The effectiveness of the proposed approaches has been demonstrated in the soft failure detection of a GPS/SDINS integrated navigation system. The simulation results show that the failure sensitivity is remarkably increased and the methods are shown to work well in detecting soft failures in the gyros and the accelerometers.

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